

1. Bernoulli-Binomial~Normal. A random sample of 400 will be selected from a population of persons registered to vote in a large community. Unknown to us is the fact that 59% of those registered favor a ballot proposal. Random variable Y will denote the number of those sampled who are in favor of the proposal. Possible values for Y are $y = 0, 1, \dots, 400$. Give a 68% interval for Y .

Solution. Since the population is large it makes no practical difference whether the sampling is with-replacement (silly) or without replacement. So samples may be regarded as independent. That is, the conditional probability for unseen sample voters' opinions are not affected by the already seen opinions of other sample voters. Therefore the conditions of the Binomial are met:

Y records the number of successes (favorable) in 400 independent Bernoulli trials with probability $p = 0.59$ of success at each trial.

We have for the Binomial that:

$$E Y = np = 400 \cdot 0.59 = 236$$

$$\text{Var } Y = npq = 400 \cdot 0.59 \cdot 0.41 = 96.76$$

$$\text{SD } Y = \sqrt{\text{Var } Y} = \sqrt{96.76} = 9.84.$$

Since we expect 236 successes and 164 failures, each of which is 10 or more, we have met the conditions given in your book for application of the normal approximation for the distribution of Y . So a 68% interval for Y is:

$$\sim [\text{Mean} \pm \text{SD}] = [236 - 9.84, 236 + 9.84] \sim [226, 246]$$

Notice the important fact that the SD of Y is relatively small when compared with the mean. It is as though the randomness is vanishing as the sample size increases. So around 68% samples of 400 voters have Y fall in the range $[226, 246]$ if $p = 0.59$.

2. Poisson~Normal. We average around 6.2 raisins per cookie. If raisins are randomly mixed into the cookie dough we suspect that the Poisson model for counts of rare events may apply. Suppose it does. Let Y = the number of raisins in a randomly selected cookie. Give a 95% interval for Y .

Solution. From experience we have $E Y \sim 6.2$. For the Poisson $\text{Var } Y = E Y = 6.2$ also! This is not a misprint since the Poisson typically arises as itself

an approximation of the Binomial when n is large and p is near zero (counts of many attempts at rare events) where we have:

$$E Y = np \text{ (around 6.2)}$$

$$\text{Var } Y = npq \sim np \sim 6.2 \text{ (since } q \sim 1 \text{ if } p \sim 0 \text{).}$$

$$\text{SD } Y = \text{root}(6.2) \sim 2.5$$

Just remember that for the Poisson the variance is the same as the mean and the SD for the Poisson is the square root of the mean! Since the mean 6.2 is at least 3 we meet the conditions for a useful approximation of the Poisson by the normal. So a 95% interval for the number of raisins in a cookie is:

$$[\text{mean} \pm 2 \text{ SD}] = [6.2 - 2 \cdot 2.5, 6.2 + 2 \cdot 2.5] \sim [1, 11].$$

The expressions:

np

npq

mean

$\text{root}(\text{mean})$

will appear on Exam 1 without explanation. You will need to know their connections with the above.